SCALING RADIAL-SECTOR FFAG ACCELERATORS

– A HIGGS FACTORY LATTICE EXAMPLE –

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HEMC Workshop

October 30, 1999

A collaboration with Weishi Wan, Carol Johnstone and Fred Mills.

YSICAL REVIEW VOLUME 103, NUMBER 6 SEPTEMBER 15, 1956

Fixed-Field Alternating-Gradient Particle Accelerators*

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(Received June 6, 1956)

It is possible, by using alternating-gradient focusing, to design circular accelerators with magnetic guide fields which are constant in time, and which can accommodate stable orbits at all energies from injection to output energy. Such accelerators are in some respects simpler to construct and operate, and moreover, they show promise of greater output currents than conventional synchrotrons and synchrocyclotrons. Two important types of magnetic field patterns are described, the radial-sector and spiral-sector patterns, the former being easier to understand and simpler to construct, the latter resulting in a much smaller accelerator for a given energy. A theory of orbits in fixed-field alternating-gradient accelerators has been worked out in linear approximation, which yields approximate general relationships between machine parameters, as well as more accurate formulas which can be used for design purposes. There are promising applications of these principles to the design of fixed-field synchrotrons, betatrons, and high-energy cyclotrons.

Features of FFAG Accelerators

Advantages

- Large energy acceptance
- Small closed-orbit radial spread
  - uniform with azimuth
- Constant orbit properties for all energies

Disadvantages

- Large circumference
- Non-linear magnetic field profile

Muon Accelerator Example

A machine to accelerate muons from 16 GeV to 64 GeV in about 30 turns with 15 MV/m rf cavities

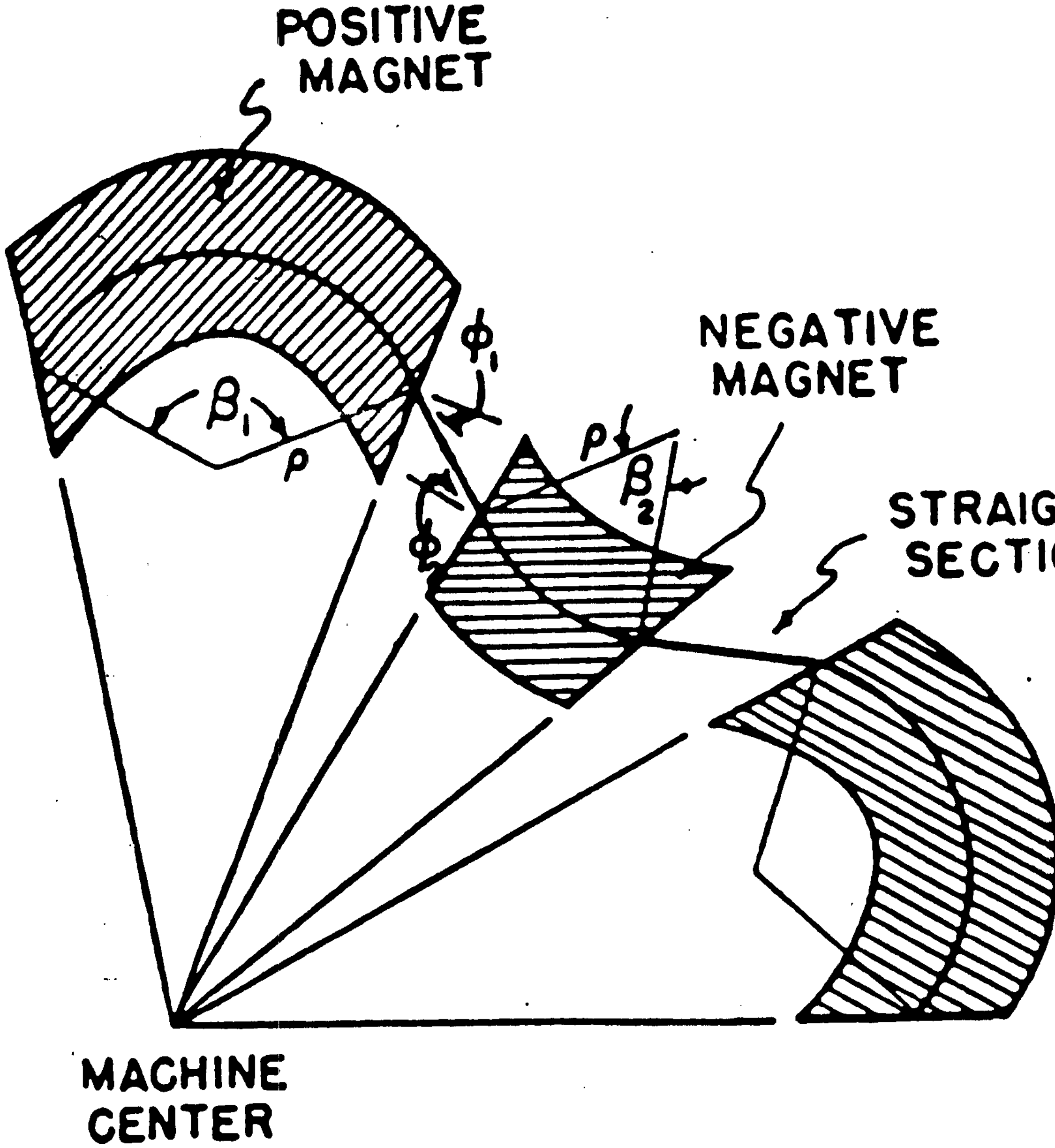


FIG. 7. Equilibrium orbit notation for radial sectors with straight sections.

FFAG concepts

The fixed field alternating gradient (FFAG) idea was invented at MURA in the 1950s. The idea is to make a ring using combined-function bend magnets with field profiles, curvatures, and edge angles chosen to make closely-spaced parallel closed orbits over a wide energy range. The linear orbit properties - tunes, and beta functions - are constant with energy.

Different types of FFAG machines were devised. The type used for this muon accelerator example is a Radial Sector FFAG. Each periodic cell of such a machine contains one F and one D bending magnet and two drift spaces. The magnetic fields in these magnets alternate in sign, but the gradient strengths always increase outward from the center of the ring.

The values of the fields and gradients in the F and D magnets have the same magnitudes. The radial dependence of the field strength is r to the power n, where r is the local radius of curvature and n = -(r/B)(dB/dr). The field index n has the same magnitude but its sign is positive and negative in the F and D magnets respectively.

The F and D magnets bend in and out respectively, so to give net inward bending, the Fs are made longer than the Ds.

Assume: $\rho = \rho_F = \rho_D$

$|n| = -n_F = n_D$

$l_F = \frac{3}{2} l_D \rightarrow$ circ. factor $= 5$

cell Tunes $\mu_x = 5/6\,\pi$, $\mu_y = 1/6\,\pi$

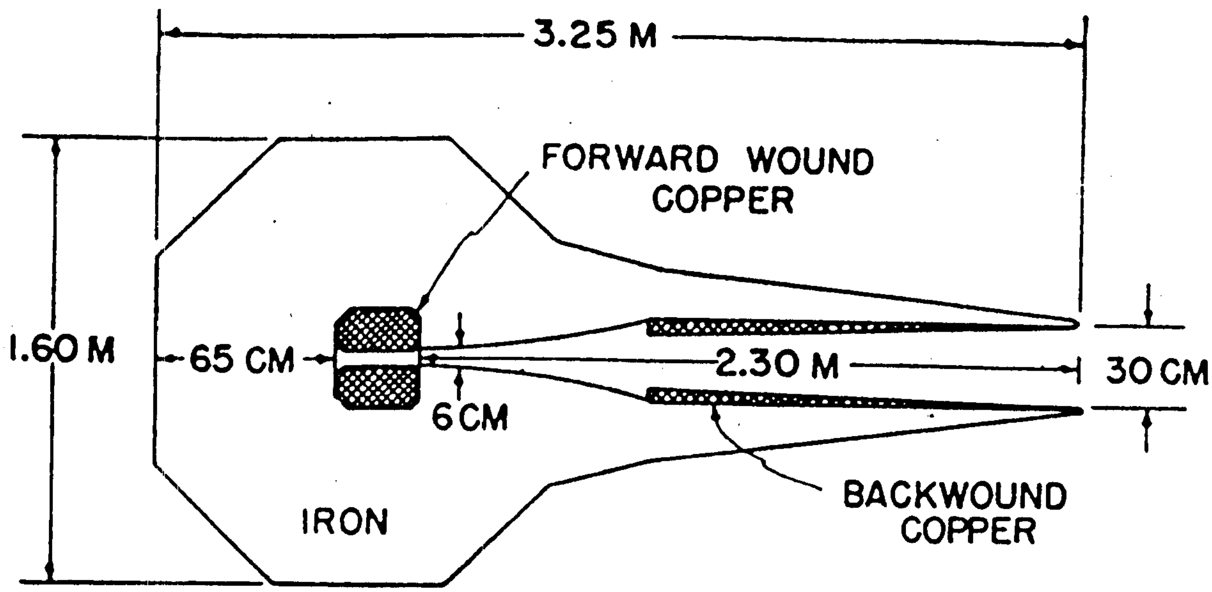


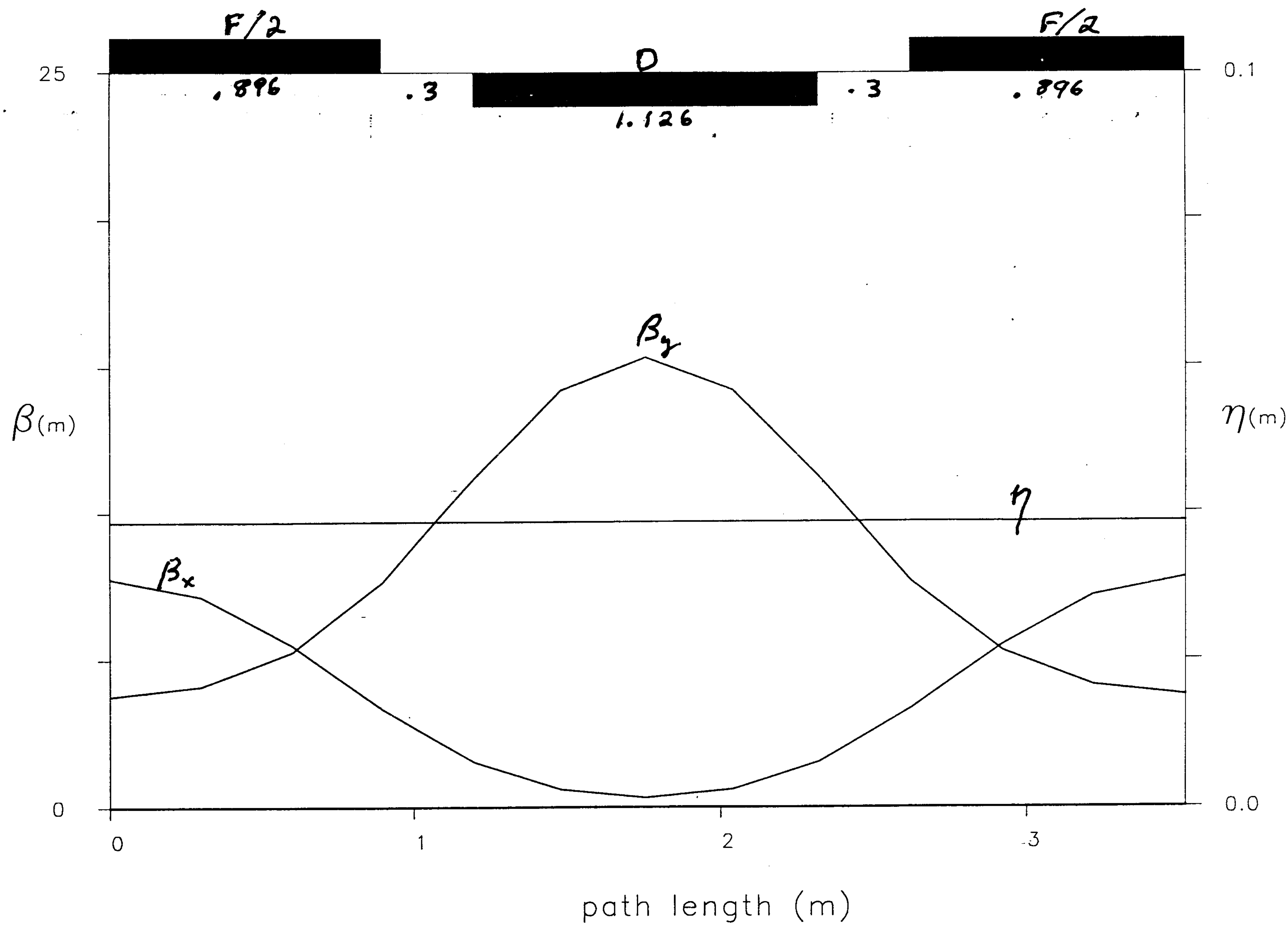
FIG. 12. Cross section of radial-sector magnet and coils.

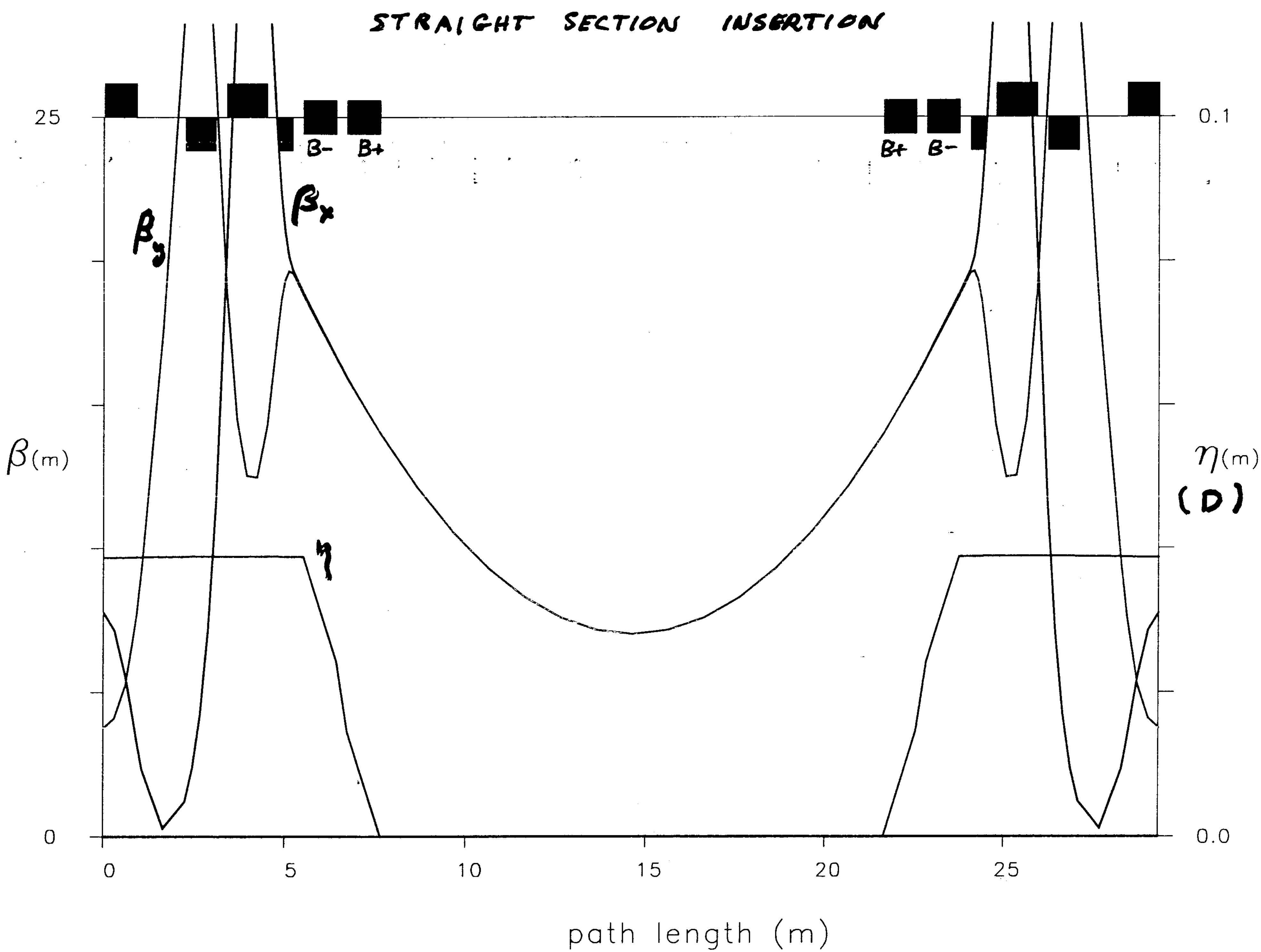
ıere $w=\lambda/2\pi$ and λ is the radial separation betwee jacent ridges in units of the radius.

Parameters for a 20-Bev ring magnet will be derive ing this smooth-approximation result and the co ion $\sigma=2\pi\nu/N<\pi$, the stability limit for a H uation. Later the alteration of these paramete sulting from exact solution of the linearized diffe tial equation by the use of the Illiac digital comput ll be shown.

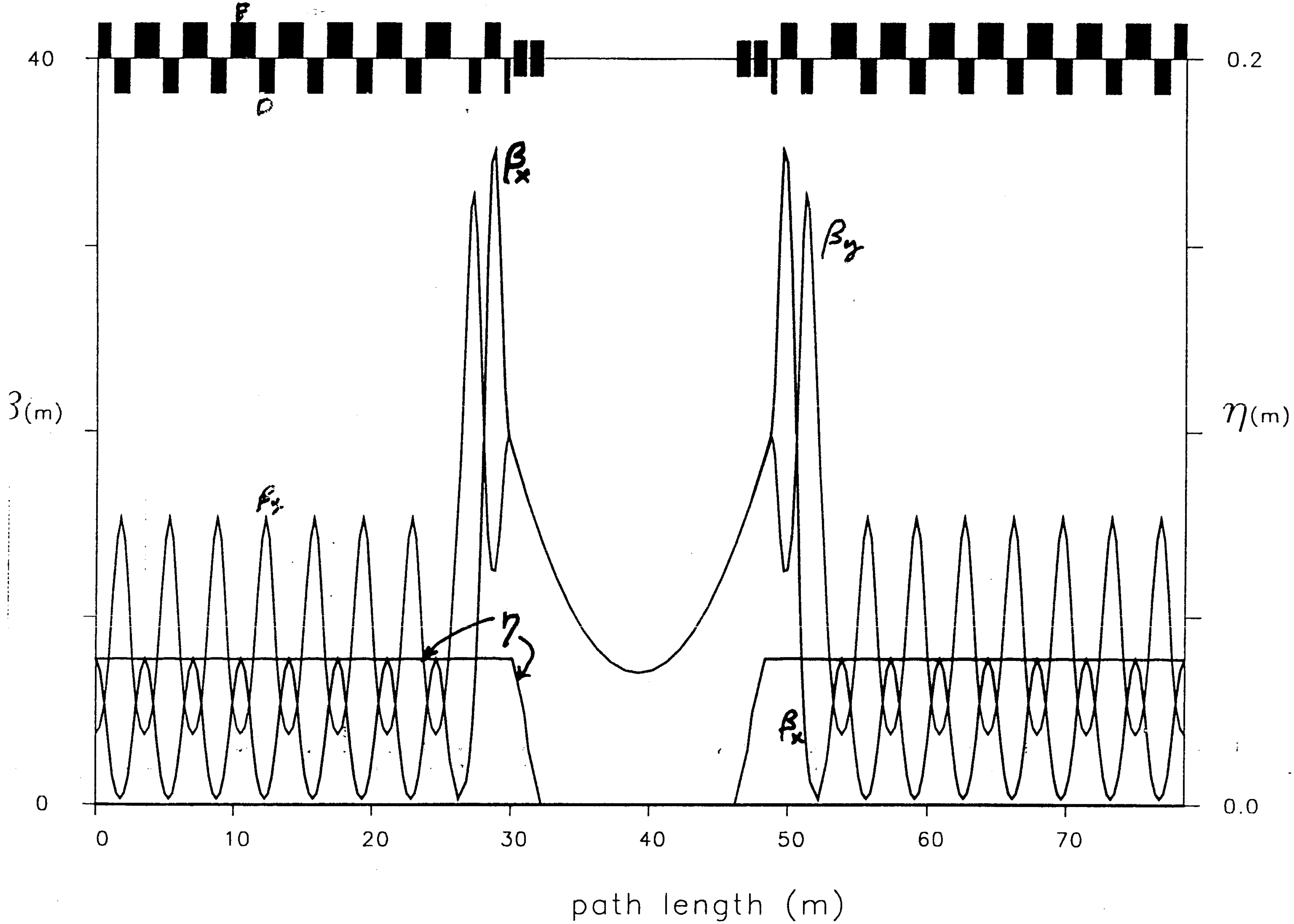
We can choose from many types of injectors—line celerators of 50 Mev, cyclotrons, or, for much low ergy, Van de Graaf electrostatic accelerators. For th rpose of this example, suppose we choose an extren se in which the ring magnet is able to hold orbits Mev injected protons at its inside rim and orbits -Bev protons at its outside rim. We can choo

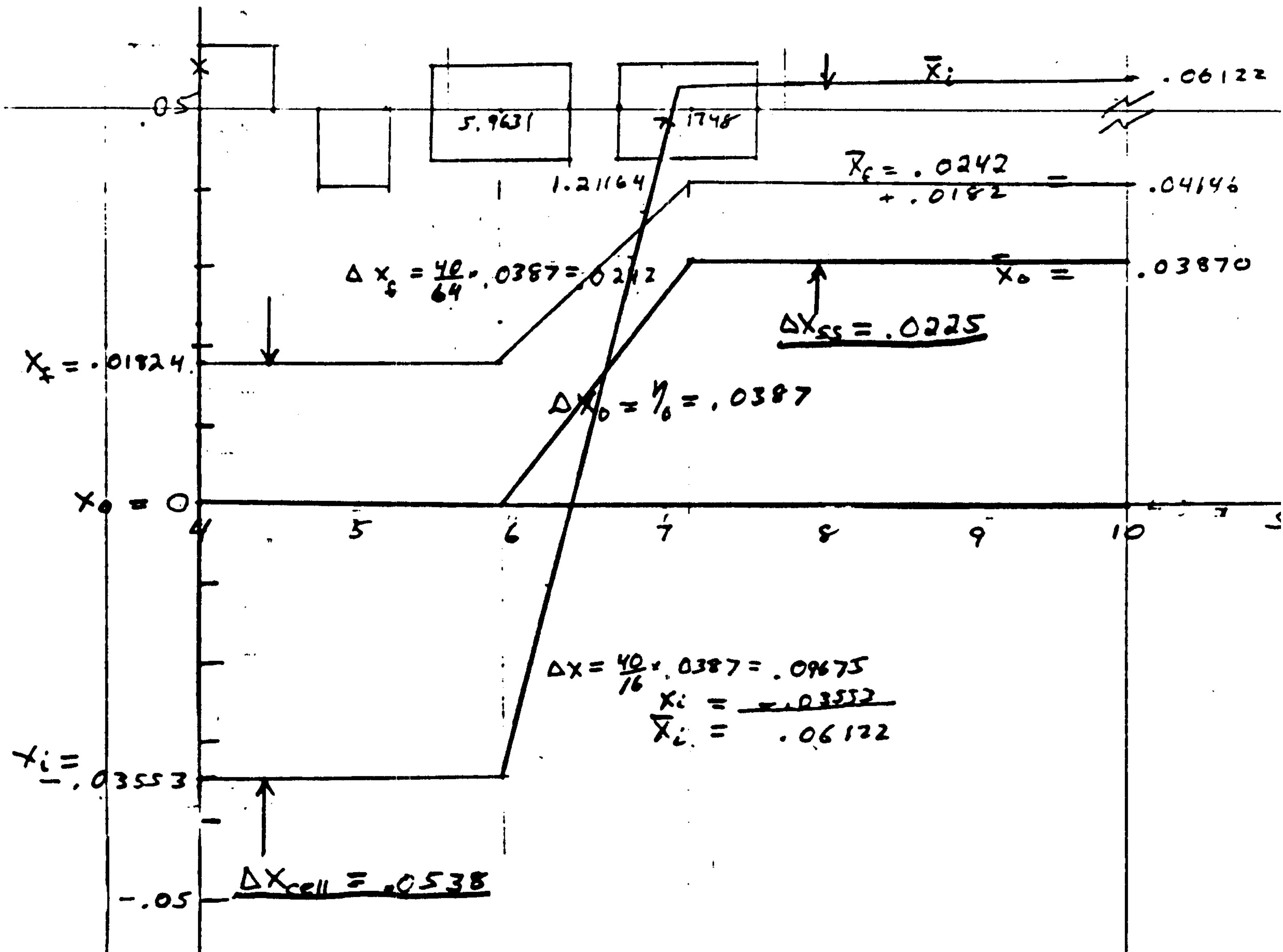
ARC CELL FOR MUON ACCELERATOR




STRAIGHT SECTION INSERTION
25
0
β(m)
η(m)
(D)
0.1
0.0
B-
B+
B+
B-
β_y
β_x
η
0
5
10
15
20
25
path length (m)

SECTOR (1/16 of ring) WITH FODO CELLS
AND STRAIGHT SECTION INSERTION





⇒ Reduce strength of bend 25%:

$\bar{X}_o = .0387 \times .25 = .0290 = \Delta X_o$

.02322

$\Delta X_f = \frac{40}{64} \cdot \Delta X_o = .0181, \quad X_f = .0182, \quad \bar{X}_f = .0363$

$\Delta X_i = \frac{40}{16} \cdot .029 = .0726, \quad X_i = -.0355, \quad \bar{X}_i = .0370$

$\Delta \bar{X}_{ss} = .037 - .029 = 0.008$

Momentum Compaction

The ring now has $\alpha_p = \frac{\Delta C}{C} / \frac{\Delta p}{p} = \underline{0.17 \times 10^{-3}}$

$\Delta p/p$ covers the range ± 0.6 or 1.2, so $\frac{\Delta C}{C} = 2 \times 10^{-4}$

$C = 1256$ so $\Delta C = 0.256$ m.

We should make $\Delta C \ll \lambda_{RF}$.

$\underline{f_{RF}} = \underline{300 \text{ MHz}} = 10^8 \text{ sec}^{-1} = 1/T_{RF}$

$C = \lambda_{RF}/T_{RF} = \lambda_{RF} f_{RF}$, $\lambda_{RF} = C/f_{RF} = 3$ m

But phase slip will be something like $\underline{\Sigma_i (\Delta C)_i / \lambda}$ where i turn #

If $\Delta E = 1$ GeV/turn, done over 48 turns, $\overline{\Delta C} \sim 0.1$ m per turn

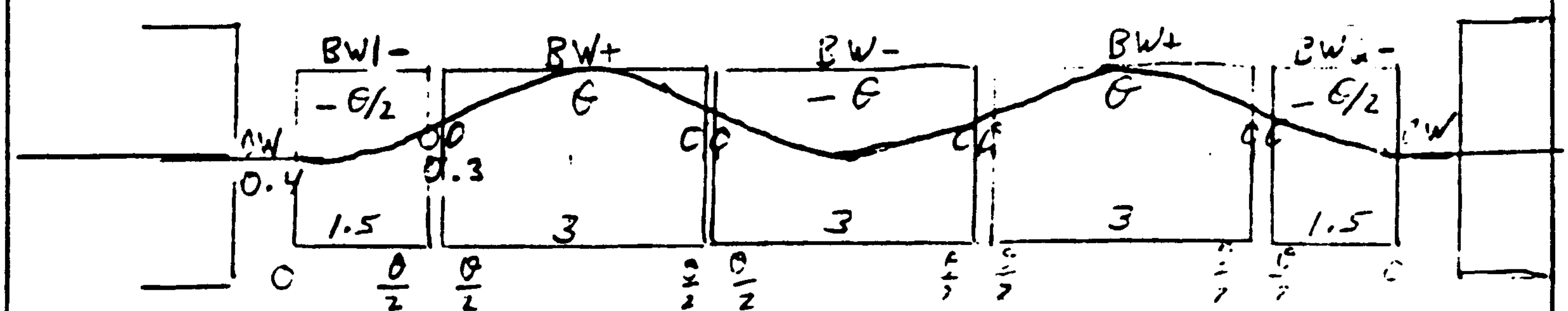
$\underline{\Delta \phi} \sim (50 \cdot 0.1\text{m} / 3\text{m})\, 360° \sim \underline{600°}$ Total phase slip for 50 turns

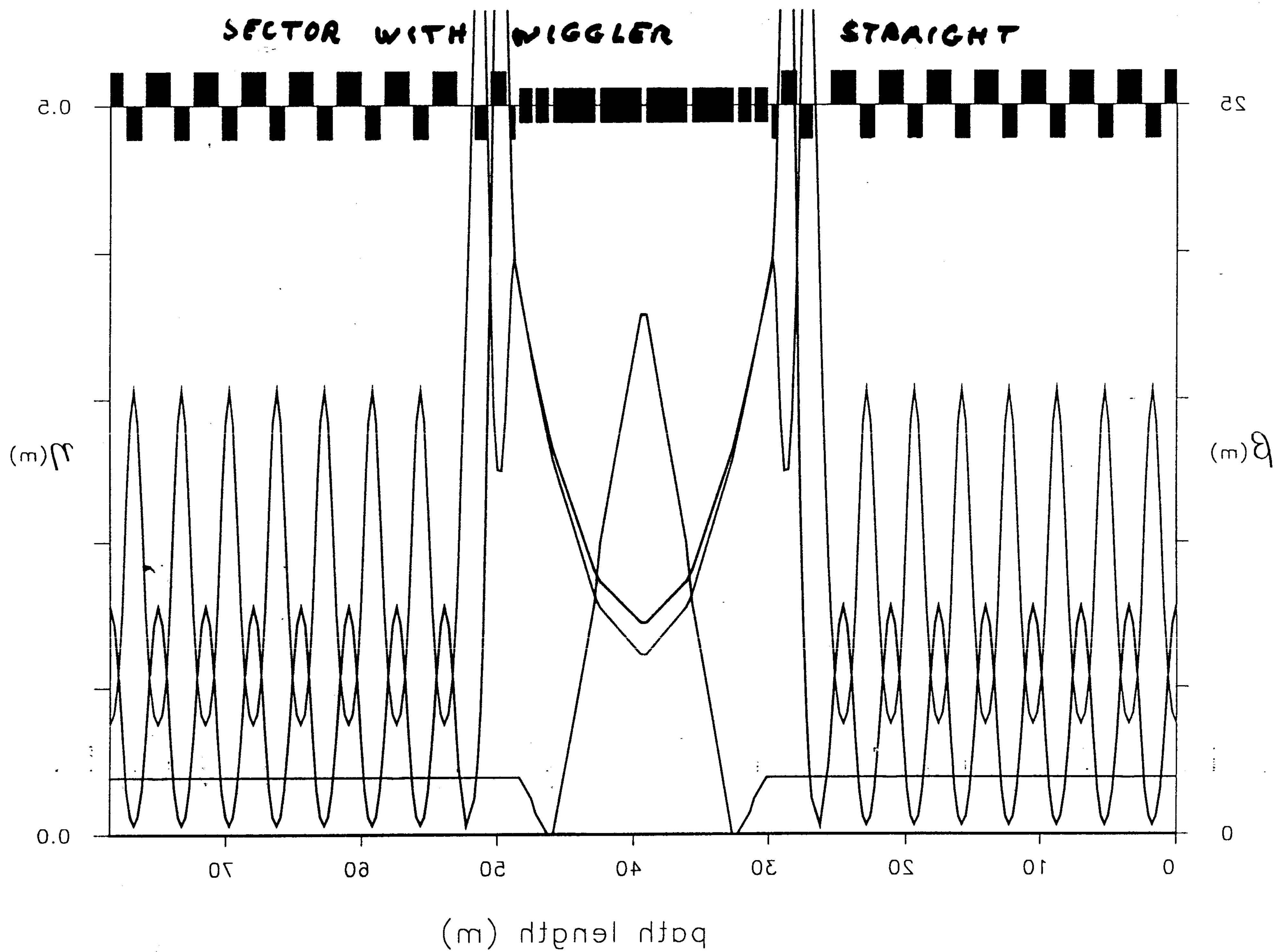
$\therefore$ Should reduce α_p to less than 10^{-5}

__Cures__ :
- (i) vary rf frequency during acceleration
- (ii) reduce α_p by using wigglers

__Wigglers__

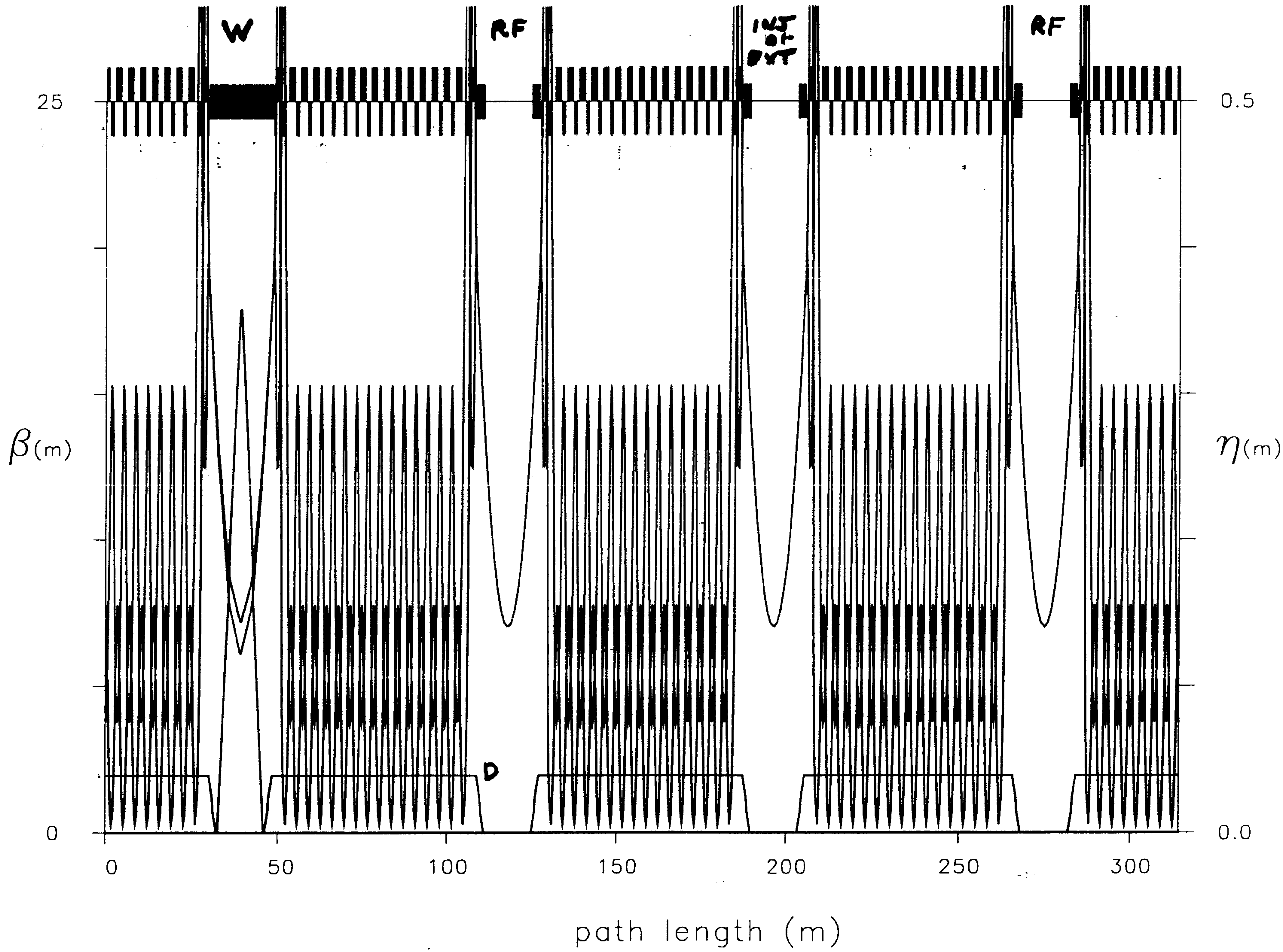
Allocation of straights: rf (8), injection/extraction (4), wigglers (4)





SECTOR WITH WIGGLER
STRAIGHT
0.5
0.0
η(m)
25
0
β(m)
0
10
20
30
40
50
60
70
path length (m)

RING QUADRANT



Arc Cell Parameters at 40 Gev Central Energy

Rigidity	BRo	133.4	T-m
Magnetic field	Bo	4.86	T
Gradient	Go	125	T/m
	Ko	0.937	m-2
Field index	n	707	
Radius of curvature	ro	27.47	m
Cell length	Lc	3.51	m
Length of F magnet	Lf	1.79	m
Length of D magnet	Ld	1.13	m
Cell tunes	mux	0.416	
	muy	0.084	
Maximum beta values	betax	7.8	m
	betay	15.3	m

Ring and Insertion Parameters

Circumference	C	1356	m
Insertion length-total	Lins	29.3	m
Insertion length-free magnet	Lif	14	m
Number of sectors	Nsec	16	
Number of superperiods	Nsp	4	
Number of insertions	Nins	16	
Ring tunes	nux	112.81	
	nuy	25.90	
Maximum betas	betax	35.1	m
	betay	32.7	m
Momentum compaction	alphap	-0.7E-08	

Performance Parameters

Injection energy	Ei	16	GeV
Central energy	Eo	40	GeV
Extraction energy	Ef	64	GeV
Radial displacements	xi	-3.6	cm
	xo	0.0	
	xf	1.8	cm
Radial spread in straights	dxs	0.8	cm